Technical Comments

Comments on "An Analytical and Experimental Study for Surface Heat Flux Determination"

James V. Beck*
Sandia Laboratories, Albuquerque, N. Mex.

THE authors[1] present an interesting paper that has both analytical and experimental components. They, however, make some questionable statements. It is necessary to refer to a previous method by the present author because one of the objectives of the paper was to compare the proposed analytical method with the existing method.[2]

1) The title of the paper states that it is about the determination of the surface heat flux, but all the figures relate to the determination of the surface temperatures. The determination of the surface temperature is less difficult than for the surface heat flux because it is less sensitive to measurement errors. Hence, conclusions (such as the need of future temperatures) based on figures in Ref. 1 may not be valid for surface heat flux determination. Table 2 presents a summary of results for the heat flux for an analytical case, but neither the accuracy of the data nor the dimensionless times are stated. Table 3, also for heat flux, is for experimental data; if this case involves an *unknown* heat flux and different errors in each thermocouple, validity of the average error comparison is in question.

2) The authors[1] propose using a *single* interior thermocouple with calculational time steps equal to the experimentally measured values and to match the measured temperatures very closely. The method of Ref. 2 is more general in that more than one interior thermocouple can be used, finer calculational time steps can be used than measured values, and additional future temperatures can be used. These are all desirable features but one does not have to use them.

For the conditions stated above, the method of Ref. 2 is reduced to taking only the first term in the sum of squares (i.e., no additional future temperatures),

$$F=(T_r'-T_n^*)^2 \tag{1}$$

where T_r' is the calculated temperature at node r and T_n^* is the corresponding measured temperature. If the thermal properties in the time interval between the present time and the future time (denoted by the prime on T) are assumed to be constant with time, the calculation for this interval is *linear* in terms of the heat flux. (This assumption is used in the derivation of Eq. (8) in Ref. 1) Then temperature T_r' can be given exactly by

$$T_r'=T_r+\frac{\partial T_r'}{\partial \dot{q}_{\text{net}}^*}\left(\dot{q}_{\text{net}}^*-\dot{q}_{\text{net}}\right) \tag{2}$$

where T_r is the future temperature using the present heat flux $\dot{q}_{\text{net}}$, and $\dot{q}_{\text{net}}^*$ is the unknown future surface heat flux.

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*Member Technical Staff, Thermal Test and Analysis Division.

In the present case F is set equal to zero which results in the equation for $\dot{q}_{\text{net}}^*$ of

$$\dot{q}_{\text{net}}^*=\dot{q}_{\text{net}}+\frac{(T_n^*-T_r)}{\partial T_r'/\partial \dot{q}_{\text{net}}^*} \tag{3}$$

No iterations are needed. The sensitivity coefficient[3] is given simply by

$$\frac{\partial T_r'}{\partial \dot{q}_{\text{net}}^*}=\frac{T_r'[(1+\epsilon)\dot{q}_{\text{net}}]-T_r'(\dot{q}_{\text{net}})}{\epsilon \dot{q}_{\text{net}}} \tag{4}$$

which implies two separate calculations for T_r' — one using $\dot{q}_{\text{net}}^*=\dot{q}_{\text{net}}$, the previously found value, and $\dot{q}_{\text{net}}^*=(1+\epsilon)\dot{q}_{\text{net}}$, where ϵ is a convenient value such as 1. (Due to the assumption of constant properties for one time interval and the resulting linearity, ϵ does not have to be a small value.) For the assumptions used by the authors, the existing method[2] reduces to simply Eqs. (3) and (4). This procedure is simpler and more straightforward than the proposed iterative equations, i.e., authors' Eq. (8)[1] followed by the method of false position [Eq. (9)] *and* a further quadratic equation [Eq. (11)]. Notice that *no* convergence check is needed such as Eqs. (10) and (12) of Ref. 1 for the above assumptions in the method of Ref. 2.

In the efficiency comparison given by the authors it is only fair to compare on the same basis, that is, no future temperature ($r=1$) and calculational time steps equal to measured values ($m=1$). On this basis only three evaluations of the tridiagonal matrix would be required and *no* iterations would be required. The authors' method[1] cannot use fewer evaluations and could require many more. Hence, the present author does not agree with the iteration count given in Table 2.

3a) The authors state several times that future temperatures gave no advantage when used in the Beck method.[1] While it is true that future temperatures may not be needed[4,5] for large dimensionless times ($\alpha\Delta t/E^2>0.5$), they are usually needed for small dimensionless times ($\alpha\Delta t/E^2\approx 0.05$) as discussed in point 3b below. Though the authors do mention the dimensionless time, it is difficult to relate values with their curves. Furthermore, whenever the authors smooth data before analysis, *future* T *information is used.*

Using orthogonal polynomials and least-squares to piecewise smooth the data is reasonable but there are problems. For example, what order of polynomial should be used and how many data points should be included in the least-squares procedure? The use of least-squares directly on the data as in the method of the Ref. 2 (unlike the authors') has advantages including a statistical basis[4] and direct extension to multiple thermocouples in both one and two dimensional cases.

3b) The implicit method of solution of the heat conduction equation (when satisfying the measured temperature exactly, as done by the authors) implies time differences of temperature and heat flux at the location of the thermocouple.[6] The differences of T and $\dot{q}$ as the number of nodes increases approach a similar form as the exact solution given by Burggraf,[7] which for omission of the $\dot{q}$ terms at the thermocouple is

$$\dot{q}_{\text{net}}=\frac{k}{E}\sum_{i=1}^{\infty}\frac{1}{(2i-1)!}\frac{\mathrm{d}^i T_n^*}{\mathrm{d}\tau^i} \tag{5}$$

where T_n^* is the measured temperature and $\tau=\alpha t/E^2$. For the implicit method the time derivative in Eq. (5) is replaced by

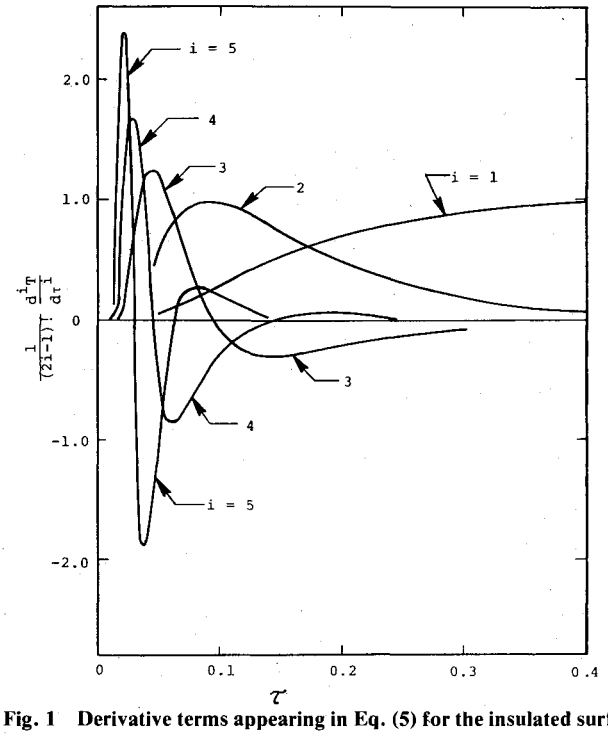

Fig. 1 Derivative terms appearing in Eq. (5) for the insulated surface of a plate heated with a constant flux on one side and insulated on the other.

backward time differences of T_n^* divided by $\Delta\tau$ to the i power.

In order to obtain insight into the relative values of each of the terms in the above summation, the values for the first five i's are plotted in Fig. 1 for a plate heated by a constant heat flux on one side and insulated on the other; the curves are for the insulated surface. Notice that the first derivative term is the only important one for $\tau > 0.4$ but, as the time is decreased, more and *higher* order terms become important. After *any* rapid change in surface heat flux, similar behavior of the derivatives would occur.

As time steps become small ($\Delta\tau = 0.02$ is mentioned[1]), the authors' method implies approximating high-order time derivatives which can be very difficult to do accurately with temperature *measurements* because high-order differences are involved.

The method of Ref. 2 was developed for cases in which the surface flux might change abruptly with time and the maximum information regarding the surface conditions was desired. These conditions require the minimum possible time step. By utilizing future temperature measurements, accurate heat fluxes can be predicted utilizing unsmoothed data (of sufficient accuracy[4]) for small time steps. Additional future measurements may not be needed for extremely accurate data and/or relatively large time steps.

A conclusion of Ref. 1 was that no advantage was gained using the method of Ref. 2 with future temperatures. While this could be true for determining the surface temperature for smoothed data, smoothly varying heat fluxes, and/or large time steps, it is not true in general for determining the surface heat flux.

References

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Reply by Authors to James V. Beck

S.D. Williams*
Lockheed Electronics Company, Inc., Houston, Texas
and
Donald M. Curry†
NASA Lyndon B. Johnson Space Center, Houston, Texas

SINCE one of the objectives of our paper[1] was to compare Beck's method with the method we proposed, the comments by Dr. Beck are appreciated. The following remarks are offered in response to his comments.

1) In solving for the net heating rate, the surface temperature is obtained by either method, that of Ref. 1 or Beck's.[2] Thus, if the converged value for the net heating rate is known, so is the surface temperature. The surface heat flux, on the other hand, is known only if the surface properties, such as emmisivity, are known. In dealing with experimental data, the emissivity is usually only approximated and the surface heating rate is given by

$$\dot{q}_{\text{conv}} = \dot{q}_{\text{net}} + \sigma\epsilon(T_s^4 - T_0^4)$$

where $\dot{q}_{\text{conv}}$ is the convective heating rate, q_{net} is the net heating rate, σ is the Stefan-Boltzmann constant, ϵ is the emissivity, T_s is the surface temperature, and T_0 is the surface radiation sink temperature. In any case, it is more valid to discuss surface temperature or net heating than the surface heating rate. Neither is more or less difficult.

The information relating to properties for the linear problem was deleted due to the recommendation of the referees. For the linear model[1] the thermal model consists of 5.08 cm (2 in.) of aluminum with a thermocouple on the backwall. Since this is a linear problem, the surface emittance was set to zero and constant properties were used. The thermal properties were: density = 2851.29 kg/m^3 (178 lb/ft^3); specific heat = 836.80 J/kg-K (0.2 Btu/lb$_m$-°R); and conductivity = 145.28 W/m-K (84 Btu/ft-h-°R). An initial temperature of 294.44 K (530°R) was used, and the backwall was insulated; i.e., adiabatic.

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*Senior Engineer, Applied Mechanics Department.
†Senior Engineer, Structures and Mechanics Division.